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# THE BOUC-WEN MODEL FOR MAGNETO-RHEOLOGICAL DAMPERS: A SENSITIVITY ANALYSIS

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## INTRODUCTION

Magneto-rheological (MR) dampers are semi-active devices that are able to reversibly change the viscosity of their fluids by exposing them to a controlled magnetic field. These devices are one of the most promising ones for practical applications in vibration isolation and suppression problems [5], among other [1].

The main characteristics of the MR dampers that have attracted the attention of researches in the past few years are their inherent stability and versatility, and their capacity of providing performances comparable to that of active devices, however, with much lower power demand [2]. Nonetheless, the highly nonlinear and hysteretic dynamic behavior of the MR dampers requires an accurate mathematical model for their practical application succeed.

The present work deals with the inverse problem of parameter estimation in the Bouc-Wen model for MR dampers [4]. A sensitivity analysis, based on the Finite Difference Method [3], was carried out in order to gain some insights about the parameter estimation at hand and for experiment design purposes. The parameter identification problem was solved considering the gradient based Levenberg-Marquardt method.

## PROBLEM FORMULATION

The MR damper force in the Bouc-Wen model is given by

$$\begin{aligned} u(t) &= c_0\dot{q}(t) + k_0 q(t) + z(t) + u_0 \\ \dot{z}(t) &= -\gamma|\dot{q}(t)|\,z(t)\,|z(t)|^{n-1} - \beta\dot{q}(t)|z(t)|^n + A\dot{q}(t) \end{aligned} \quad (1)$$

where $q(t)$ is the displacement at the end of the MR damper, $c_0$ is the plastic damping coefficient, $k_0$ and $u_0$ account for the effects of the accumulator in the damper, $z(t)$ is an internal variable and the parameters γ, β, $A$ and $n$ control the shape of the hysteresis curve.

The present inverse problem of parameter estimation can then be implicity stated as the minimization problem

$$\min_{\mathbf{x}} Q(\mathbf{x}) = \min_{\mathbf{x}} \frac{1}{2} \sum_{i=i}^{N_t} (u_i(\mathbf{x}) - u_{exp,i})^2 \quad (2)$$

where $\mathbf{x}$ is the vector of unknown parameters, $N_t$ is the total number of time samples considered in the estimation process and $u_i$ and $u_{exp,i}$ stand, respectively, for the analytical and experimental damper force at the time instant $t_i$.

In order to solve the parameter identification problem in Eq. (2), the Levenberg-Marquardt method was performed using the experimental data: force, displacement and velocity at the end of the MR damper.

The scaled up sensitivity matrix $\mathcal{X}$ was determined through the computation of the following sensitivity coefficients

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$$\mathcal{X}_{ij} = x_j \frac{\partial u_i(\mathbf{x})}{\partial x_j} \simeq x_j \frac{u_i([1+\varepsilon]x_j) - u_i([1-\varepsilon]x_j)}{2\varepsilon x_j} \qquad (3)$$

where $\varepsilon$ represents a small increment in the parameter value $x_j$.

## NUMERICAL RESULTS

The scaled up sensitivity curves with respect to $\gamma$, $\beta$, $A$ and $n$ are depicted in Fig. 1, for a sinusoidal displacement $q(t)$ with amplitude $0.015m$ and frequency $2.5Hz$. From Fig. 1, it can be clearly seen that these sensitivity curves are linearly dependent during most part of the time. Hence, it may not be possible to distinguish the effects of the corresponding parameters on the system response. Consequently, for the experimental data considered, the solution of the parameter estimation problem may not be unique. It is worth noting the relatively greater amplitude of the sensitivity $\mathcal{X}_n(t)$.

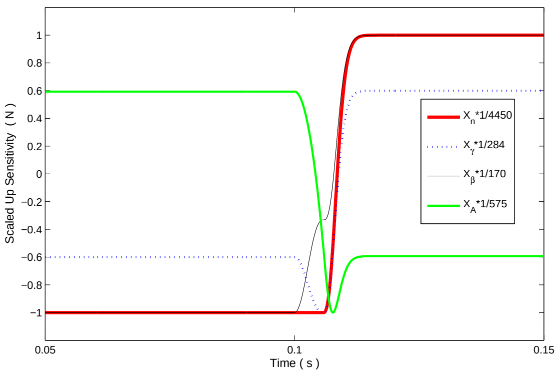


Figure 1. SCALED UP SENSITIVITY CURVES.

Therefore, a sensitivity analysis, based on graphical procedures, was carried out for experiment design purposes. The prime aim was increase the range where the sensitivity curves are not linearly dependent. This was achieved when, for instance, sinusoidal excitations with lower frequencies were considered.

Results for the parameter estimation problem are shown in Table. 1 for three cases. Case 1: sinusoidal displacement with frequency $2.5Hz$ and without noise in the damper force; Case 2: sinusoidal displacement with frequency $2.5Hz$ and with $40dB$ of signal to noise ration (SNR) in the damper force; Case 3: sinusoidal displacement with frequency $0.5Hz$ and $40dB$ of SNR in the damper force. In Table 1, $\mathbf{x}_r^0$ and $\mathbf{x}_r$ are, respectively, the initial guess and the estimated parameters relative to the exact parameter values.

Table 1. PARAMETER ESTIMATION RESULTS.

| | | Case 1 | Case 2 | Case 3 |
|---|---|---|---|---|
| Parameter | $\mathbf{x}_r^0$ | | $\mathbf{x}_r$ | |
| $c_0$ | 0.8 | 1.0 | 0.999 | 1.000 |
| $k_0$ | 0.8 | 1.0 | 0.998 | 1.005 |
| $u_0$ | 0.8 | 1.0 | 0.998 | 1.001 |
| $\gamma$ | 0.8 | 1.0 | 0.424 | 1.032 |
| $\beta$ | 0.8 | 1.0 | 0.403 | 1.038 |
| $A$ | 0.8 | 1.0 | 0.987 | 1.002 |
| $n$ | 0.8 | 1.0 | 1.067 | 0.998 |

## CONCLUSIONS

Exhaustive numerical results yielded the conclusion that the greater difficulties in the parameter estimation are related with the parameters $\gamma$ and $\beta$. This corroborates with the linearly dependence between the corresponding sensitivity curves during most part of the time. Sensitivity analysis, based on graphical interpretation of the sensitivity curves, revealed that sinusoidal excitations with lower frequencies increase the time ranges were there are no linear dependence between these curves, facilitating, then, the parameter estimation procedure.

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